A Nonlinear Generalized Thermoelasticity Model of Temperature-Dependent Materials Using Finite Element Method

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Abstract

In this article, a general finite element method (FEM) is proposed to analyze transient phenomena in a thermoelastic model in the context of the theory of generalized thermoelasticity with one relaxation time. The exact solution of the nonlinear model of the thermal shock problem of a generalized thermoelastic half-space of temperature-dependent materials exists only for very special and simple initial- and boundary problems. In view of calculating general problems, a numerical solution technique is to be used. For this reason, the FEM is chosen. The results for the temperature increment, the stress components, and the displacement component are illustrated graphically with some comparisons.

Keywords

Finite element method Generalized thermoelasticity
Temperature-dependent materials

List of Symbols

λ, μ

Lame's constants

ρ

Density

C_E

Specific heat at constant strain

t

Time

T

Temperature

T_0

Reference temperature

θ

$(T - T_0)$, temperature increment, $\frac{|T-T_0|}{T_0} \ll 1$

α_T

Coefficient of linear thermal expansion

γ

$\alpha_T(3\lambda + 2\mu)$

σ_{ij}

Components of the stress tensor

e_{ij}

Components of the strain tensor

u_i

Components of the displacement vector

k

Thermal conductivity

τ_0

Relaxation time

c_0

$\sqrt{\frac{\lambda_0+2\mu_0}{\rho_0}}$, longitudinal wave speed

η

$\frac{\rho_0 C_E}{\kappa_0}$, thermal viscosity

ε

$\frac{\gamma_0}{\rho_0 C_E}$, dimensionless thermoelastic coupling constant

a

$\frac{\gamma_0 T_0}{\lambda_0+2\mu_0}$, dimensionless thermomechanical coupling constant

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